THE ORBIT OF NEPTUNE,

COMPUTED BY

SEARS C. WALKER, A. A. S.

AND

FORMULÆ IN THE THEORY OF NEPTUNE,

BY

BENJAMIN PEIRCE, A. A. S.

FROM THE PROCEEDINGS OF THE AMERICAN ACADEMY.

Read December, 1847.

Professor Peirce communicated to the Academy Mr. Sears C. Walker's elliptic elements of Neptune.

$$\left.\begin{aligned} \text{“}\pi &= \overset{\circ}{48}\ \overset{\prime}{21}\ \overset{\prime\prime}{2.93} \\ \Omega &= 130\ \ 4\ 35.03 \end{aligned}\right\} \text{mean equinox, Jan. 1, 1847.}$$

$$\begin{aligned} i &= \ \ 1\ 46\ 59.54. \\ e &= 0.00857741. \\ \mu &= 21^{\prime\prime}.55448. \\ M &= 328^{\circ}\ 31^{\prime}\ 56^{\prime\prime}.36, \text{ mean noon, Greenwich, Jan. 1, 1847.} \\ T &= 164.6181 \text{ tropical years.} \end{aligned}$$

“The normal elliptic places of Neptune, derived from the discussion of 689 observations, European and American (including the two ancient observations of Lalande), were as follows: —

$$\begin{array}{rrlrrrrl} & \text{d.} & & \circ & \prime & \prime\prime & \prime\prime & \\ \text{I.} - & 18864 = & \text{May 9th, 1795,} & 215 & 48 & 7.68 & -\ \ 44.1 & (r\ \ \ \ - 30.28778). \\ \text{II.} - & 134 = & \text{Aug. 20th, 1846,} & 326 & 45 & 30.83 & -\ \ \ 1.2 & (r^{\prime}\ \ \ - 29.99256). \\ \text{III.} - & 55 = & \text{Nov. 7th, 1846,} & 327 & 13 & 58.57 & - 227.6 & (r^{\prime\prime}\ \ - 29.99256). \\ \text{IV.} + & 95 = & \text{April 6th, 1847,} & 328 & 8 & 0.67 & + 163.8 & (r^{\prime\prime\prime}\ - 29.99256). \\ \text{V.} + & 233 = & \text{Aug. 20th, 1847,} & 328 & 57 & 44.39 & +\ \ \ 1.0 & (r^{\prime\prime\prime\prime} - 29.99256). \end{array}$$

“These elliptic places were derived from Neptune's places in the instantaneous orbit, by the subtraction of the effect of the perturbations of all the other planets, as communicated to Mr. Walker by Professor Peirce in November last.

“The expressions for the heliocentric coördinates are, —

$$\begin{aligned} x &= [9.9993769]\, r \sin. (v + \overset{\circ}{138}\ \overset{\prime}{21}\ \overset{\prime\prime}{52.13}) \text{ m. eq. Jan. 1, 1847.} \\ y &= [9.9662265]\, r \sin. (v + \ \ 48\ 55\ 27.32). \\ z &= [9.5800962]\, r \sin. (v + \ \ 45\ \ \ 2\ 37.90). \end{aligned}$$

“Mr. Walker has applied to the elliptic values of v and r the perturbations $\delta\, v$ and $\delta\, r$, which Professor Peirce communicated to him, and has compared the instantaneous values with the normal right ascensions and declinations, as follows: —

	δv	δr	Obs. — Eph. ΔA.	Obs. — Eph. ΔD.
	″		″	″
I.	+ 37.60	+ 0.01207	+ 1.9	— 1.3.
III.	+ 32.10	+ 0.01608	— 2.9	— 1.2.
IV.	+ 31.29	+ 0.01497	+ 0.8	+ 2.6.
V.	+ 29.49	+ 0.01493	— 0.2	+ 0.0.

“ Mr. Walker has omitted the comparison of place II. because it is not the result of direct observation, like the rest. A closer representation might be obtained by least squares ; but Mr. Walker prefers to wait for Mr. Peirce's new values of the perturbations.”

Professor Peirce communicated his formulæ for the perturbations of Neptune's longitude and radius vector, resulting from his second approximation to the theory of Neptune. In his first approximation, Neptune's mean time of revolution was assumed to be just twice that of Uranus, and the eccentricity of Neptune's orbit was neglected. But the present approximation is based upon Mr. Walker's orbit, which has been presented to the Academy this evening, and includes all sensible terms as high as the cubes of the eccentricities.

“ The masses of the disturbing planets, and the elements of their orbits, which are adopted in this theory, are the same with those adopted by Leverrier, in his theories of Mercury and Uranus, with the exception of the mass of Uranus, which is taken from Lamont's determination by observations of the satellites.

“ The following notation is adopted in these formulæ : —

“ t = the time in Julian years from Jan. 1, 1850.

“ The mean longitude of each planet is denoted by the appropriate symbol of that planet.

“ The Longitude of the perihelion of each planet is denoted by π with the symbol of the planet subjacent.

“ The coefficient for correction of the mass of each planet is given in the usual form with the symbol of the planet subjacent.

“ The formulæ are as follows : —

“ δv = the perturbation of Neptune's true longitude =

$$
= (1 + \mu_{\text{♅}}) \left\{ \begin{array}{lrl}
+ & 174''.37 & \sin.\ (\text{♅} - \text{♆}) \\
- & 8.63 & \sin.\ 2\,(\text{♅} - \text{♆}) \\
- & 1.70 & \sin.\ 3\,(\text{♅} - \text{♆}) \\
- & 0.54 & \sin.\ 4\,(\text{♅} - \text{♆}) \\
- & 0.21 & \sin.\ 5\,(\text{♅} - \text{♆}) \\
- & 0.09 & \sin.\ 6\,(\text{♅} - \text{♆}) \\
- & 0.04 & \sin.\ 7\,(\text{♅} - \text{♆}) \\
- & 0.02 & \sin.\ 8\,(\text{♅} - \text{♆}) \\
- & 0.01 & \sin.\ 9\,(\text{♅} - \text{♆}) \\
- & 0.01 & \sin.\ 10\,(\text{♅} - \text{♆})
\end{array} \right\}
$$

$$
+ (1 + \mu_{\text{♅}}) \left\{ \begin{array}{lrl}
+ & 1''.98 & \sin.\ (\text{♅} - \pi_{\text{♆}}) \\
- & 105.20 & \sin.\ (2\,\text{♆} - \text{♅} - \pi_{\text{♆}}) \\
- & 0.09 & \sin.\ (2\,\text{♅} - \text{♆} - \pi_{\text{♆}}) \\
+ & 14.71 & \sin.\ (2\,\text{♅} - 3\,\text{♆} + \pi_{\text{♆}}) \\
- & 0.01 & \sin.\ (3\,\text{♅} - 2\,\text{♆} - \pi_{\text{♆}}) \\
- & 0.39 & \sin.\ (3\,\text{♅} - 4\,\text{♆} + \pi_{\text{♆}}) \\
- & 0.09 & \sin.\ (4\,\text{♅} - 5\,\text{♆} + \pi_{\text{♆}}) \\
- & 0.03 & \sin.\ (5\,\text{♅} - 6\,\text{♆} + \pi_{\text{♆}}) \\
- & 0.01 & \sin.\ (6\,\text{♅} - 7\,\text{♆} + \pi_{\text{♆}}) \\
- & 0.004340 & t \cos.\ (\text{♆} - \pi_{\text{♆}})
\end{array} \right\}
$$

$$
+ (1 + \mu_{\text{♅}}) \left\{ \begin{array}{lrl}
+ & 0''.23 & \sin.\ (\text{♅} - \pi_{\text{♅}}) \\
+ & 1668.30 & \sin.\ (2\,\text{♆} - \text{♅} - \pi_{\text{♅}}) \\
+ & 0.16 & \sin.\ (2\,\text{♅} - \text{♆} - \pi_{\text{♅}}) \\
- & 58.87 & \sin.\ (2\,\text{♅} - 3\,\text{♆} + \pi_{\text{♅}}) \\
- & 0.03 & \sin.\ (3\,\text{♅} - 2\,\text{♆} - \pi_{\text{♅}}) \\
+ & 1.50 & \sin.\ (3\,\text{♅} - 4\,\text{♆} + \pi_{\text{♅}}) \\
- & 0.02 & \sin.\ (4\,\text{♅} - 3\,\text{♆} - \pi_{\text{♅}}) \\
+ & 0.28 & \sin.\ (4\,\text{♅} - 5\,\text{♆} + \pi_{\text{♅}}) \\
- & 0.01 & \sin.\ (5\,\text{♅} - 4\,\text{♆} - \pi_{\text{♅}}) \\
+ & 0.10 & \sin.\ (5\,\text{♅} - 6\,\text{♆} + \pi_{\text{♅}}) \\
- & 0.01 & \sin.\ (6\,\text{♅} - 5\,\text{♆} - \pi_{\text{♅}}) \\
+ & 0.05 & \sin.\ (6\,\text{♅} - 7\,\text{♆} + \pi_{\text{♅}}) \\
+ & 0.03 & \sin.\ (7\,\text{♅} - 8\,\text{♆} + \pi_{\text{♅}}) \\
+ & 0.01 & \sin.\ (8\,\text{♅} - 9\,\text{♆} + \pi_{\text{♅}}) \\
+ & 0.018461 & t \cos.\ (\text{♆} - \pi_{\text{♅}})
\end{array} \right\}
$$

$$
+\,(1+\mu_{\text{♅}})\left\{\begin{array}{rl}
- & 0''.86 \text{ sin. } (3\,\text{⯉} - \text{♅} - 2\,\pi_{\text{⯉}}) \\
+ & 0.86 \text{ sin. } (\text{♅} - \text{⯉}) \\
- & 0.01 \text{ sin. } (4\,\text{⯉} - \text{♅} - 3\,\pi_{\text{⯉}}) \\
+ & 0.01 \text{ sin. } (\text{♅} - \pi_{\text{⯉}}) \\
+ & 13.55 \text{ sin. } (3\,\text{⯉} - \text{♅} - \pi_{\text{⯉}} - \pi_{\text{♅}}) \\
- & 13.73 \text{ sin. } (\text{♅} - \text{⯉} + \pi_{\text{♅}} - \pi_{\text{⯉}}) \\
+ & 0.14 \text{ sin. } (4\,\text{⯉} - \text{♅} - 2\,\pi_{\text{⯉}} - \pi_{\text{♅}}) \\
- & 0.14 \text{ sin. } (\text{♅} + \pi_{\text{♅}} - 2\,\pi_{\text{⯉}}) \\
- & 3.62 \text{ sin. } (4\,\text{⯉} - 2\,\text{♅} - 2\,\pi_{\text{⯉}}) \\
+ & 28.42 \text{ sin. } (4\,\text{⯉} - 2\,\text{♅} - \pi_{\text{⯉}} - \pi_{\text{♅}}) \\
- & 55.16 \text{ sin. } (4\,\text{⯉} - 2\,\text{♅} - 2\,\pi_{\text{♅}}) \\
- & 0.03 \text{ sin. } (5\,\text{⯉} - 2\,\text{♅} - 3\,\pi_{\text{⯉}}) \\
+ & 0.03 \text{ sin. } (2\,\text{♅} - 3\,\text{⯉} + \pi_{\text{⯉}}) \\
+ & 0.23 \text{ sin. } (5\,\text{⯉} - 2\,\text{♅} - 2\,\pi_{\text{⯉}} - \pi_{\text{♅}}) \\
- & 0.23 \text{ sin. } (2\,\text{♅} - 3\,\text{⯉} + \pi_{\text{♅}}) \\
- & 0.09 \text{ sin. } (2\,\text{♅} - 2\,\text{⯉} + \pi_{\text{♅}} - \pi_{\text{⯉}}) \\
- & 0.45 \text{ sin. } (5\,\text{⯉} - 2\,\text{♅} - \pi_{\text{⯉}} - 2\,\pi_{\text{♅}}) \\
+ & 0.45 \text{ sin. } (2\,\text{♅} - 3\,\text{⯉} + 2\,\pi_{\text{♅}} - \pi_{\text{⯉}})
\end{array}\right\}
$$

$$
+\,(1+\mu_{\text{♅}})\left\{\begin{array}{rl}
- & 0''.06 \text{ sin. } (6\,\text{⯉} - 3\,\text{♅} - 3\,\pi_{\text{⯉}}) \\
+ & 0.28 \text{ sin. } (3\,\text{♅} - 5\,\text{⯉} + 2\,\pi_{\text{⯉}}) \\
+ & 0.66 \text{ sin. } (6\,\text{⯉} - 3\,\text{♅} - 2\,\pi_{\text{⯉}} - \pi_{\text{♅}}) \\
- & 2.16 \text{ sin. } (3\,\text{♅} - 5\,\text{⯉} + \pi_{\text{⯉}} + \pi_{\text{♅}}) \\
- & 2.59 \text{ sin. } (6\,\text{⯉} - 3\,\text{♅} - \pi_{\text{⯉}} - 2\,\pi_{\text{♅}}) \\
+ & 4.30 \text{ sin. } (3\,\text{♅} - 5\,\text{⯉} + 2\,\pi_{\text{♅}}) \\
+ & 2.72 \text{ sin. } (6\,\text{⯉} - 3\,\text{♅} - 3\,\pi_{\text{♅}}) \\
+ & 0.85 \text{ sin. } (2\,\text{⯉} - \text{♅} + \pi_{\text{♅}} - 2\,\pi_{\text{⯉}}) \\
- & 0.56 \text{ sin. } (\text{♅} - \text{⯉} - \pi_{\text{♅}} + \pi_{\text{⯉}}) \\
- & 0.66 \text{ sin. } (2\,\text{⯉} - \text{♅} - \pi_{\text{⯉}}) \\
- & 0.03 \text{ sin. } (3\,\text{⯉} - \text{♅} - 2\,\pi_{\text{⯉}}) \\
+ & 4.38 \text{ sin. } (\text{♅} - \text{⯉}) \\
+ & 0.13 \text{ sin. } (2\,\text{⯉} - \text{♅} - \pi_{\text{♅}}) \\
+ & 0.43 \text{ sin. } (3\,\text{⯉} - \text{♅} - \pi_{\text{♅}} - \pi_{\text{⯉}}) \\
- & 0.43 \text{ sin. } (\text{♅} - \text{⯉} + \pi_{\text{♅}} - \pi_{\text{⯉}}) \\
+ & 0.24 \text{ sin. } (2\,\text{⯉} - \text{♅} - 2\,\pi_{\text{♅}} + \pi_{\text{⯉}}) \\
+ & 0.01 \text{ sin. } (3\,\text{⯉} - \text{♅} - 2\,\pi_{\text{♅}})
\end{array}\right\}
$$

$$+(1+\mu_{\text{♄}})\left\{\begin{array}{ll} + & 18''.60 \sin.(\text{♄}-\text{♆}) \\ - & 0.15 \sin. 2(\text{♄}-\text{♆}) \\ - & 0.02 \sin. 3(\text{♄}-\text{♆}) \end{array}\right\}$$

$$+(1+\mu_{\text{♄}})\left\{\begin{array}{ll} - & 0''.08 \sin.(\text{♄}-\pi_{\text{♆}}) \\ + & 0.22 \sin.(\text{♄}-2\,\text{♆}+\pi_{\text{♆}}) \\ - & 0.01 \sin.(2\,\text{♄}-3\,\text{♆}+\pi_{\text{♆}}) \\ - & 0.003404\, t \cos.(\text{♆}-\pi_{\text{♆}}) \end{array}\right\}$$

$$+(1+\mu_{\text{♄}})\left\{\begin{array}{ll} + & 0''.01 \sin.(\text{♄}-\pi_{\text{♄}}) \\ + & 0.28 \sin.(\text{♄}-2\,\text{♆}+\pi_{\text{♄}}) \\ + & 0.54 \sin.(2\,\text{♄}-\text{♆}-\pi_{\text{♄}}) \\ + & 0.02 \sin.(2\,\text{♄}-3\,\text{♆}+\pi_{\text{♄}}) \\ + & 0.009134\, t \cos.(\text{♆}-\pi_{\text{♄}}) \end{array}\right\}$$

$$+(1+\mu_{\text{♃}})\left\{\begin{array}{ll} + & 34''.09 \sin.(\text{♃}-\text{♆}) \\ - & 0.02 \sin. 2(\text{♃}-\text{♆}) \end{array}\right\}$$

$$+(1+\mu_{\text{♃}})\left\{\begin{array}{ll} - & 0''.14 \sin.(\text{♃}-\pi_{\text{♆}}) \\ + & 0.42 \sin.(\text{♃}-2\,\text{♆}+\pi_{\text{♆}}) \\ - & 0.02931\, t \cos.(\text{♆}-\pi_{\text{♆}}) \end{array}\right\}$$

$$+(1+\mu_{\text{♃}})\left\{\begin{array}{ll} + & 0''.01 \sin.(\text{♃}-2\,\text{♆}+\pi_{\text{♃}}) \\ + & 0.82 \sin.(2\,\text{♃}-\text{♆}-\pi_{\text{♃}}) \\ + & 0.003727\, t \cos.(\text{♆}-\pi_{\text{♃}}) \end{array}\right\}$$

$$+(1+\mu_{\oplus})\, 0''.02 \sin.(\oplus-\text{♆})$$

“ δr = the perturbation of Neptune’s radius vector =

$$=(1+\mu_{\text{♅}})\left\{\begin{array}{l} +0.00061 \\ -0.01328 \cos.(\text{♅}-\text{♆}) \\ +0.00097 \cos. 2(\text{♅}-\text{♆}) \\ +0.00023 \cos. 3(\text{♅}-\text{♆}) \\ +0.00008 \cos. 4(\text{♅}-\text{♆}) \\ +0.00003 \cos. 5(\text{♅}-\text{♆}) \\ +0.00001 \cos. 6(\text{♅}-\text{♆}) \\ +0.00001 \cos. 7(\text{♅}-\text{♆}) \end{array}\right\}$$

$$+(1+\mu_{♅})\left\{\begin{array}{l}
-\ 0.00012 \text{ cos. } (♅-\pi_{♆}) \\
+\ 0.00055 \text{ cos. } (2\,♆-♅-\pi_{♆}) \\
+\ 0.00001 \text{ cos. } (2\,♅-♆-\pi_{♆}) \\
-\ 0.00101 \text{ cos. } (2\,♅-3\,♆+\pi_{♆}) \\
+\ 0.00004 \text{ cos. } (3\,♅-4\,♆+\pi_{♆}) \\
+\ 0.00001 \text{ cos. } (4\,♅-5\,♆+\pi_{♆}) \\
-\ 0.000000819\ t \text{ sin. } (♆-\pi_{♆})
\end{array}\right\}$$

$$+(1+\mu_{♅})\left\{\begin{array}{l}
-\ 0.00003 \text{ cos. } (♅-\pi_{♅}) \\
-\ 0.00666 \text{ cos. } (2\,♆-♅-\pi_{♅}) \\
+\ 0.00001 \text{ cos. } (2\,♅-♆-\pi_{♅}) \\
+\ 0.00403 \text{ cos. } (2\,♅-3\,♆+\pi_{♅}) \\
-\ 0.00016 \text{ cos. } (3\,♅-4\,♆+\pi_{♅}) \\
-\ 0.00004 \text{ cos. } (4\,♅-5\,♆+\pi_{♅}) \\
-\ 0.00002 \text{ cos. } (5\,♅-6\,♆+\pi_{♅}) \\
-\ 0.00001 \text{ cos. } (6\,♅-7\,♆+\pi_{♅}) \\
+\ 0.000001362\ t \text{ sin. } (♆-\pi_{♅})
\end{array}\right\}$$

$$+(1+\mu_{♅})\left\{\begin{array}{l}
+\ 0.00005 \text{ cos. } (3\,♆-♅-\pi_{♆}-\pi_{♅}) \\
+\ 0.00003 \text{ cos. } (♅-♆+\pi_{♅}-\pi_{♆}) \\
+\ 0.00005 \text{ cos. } (4\,♆-2\,♅-2\,\pi_{♆}) \\
-\ 0.00048 \text{ cos. } (4\,♆-2\,♅-\pi_{♆}-\pi_{♅}) \\
+\ 0.00095 \text{ cos. } (4\,♆-2\,♅-2\,\pi_{♅})
\end{array}\right\}$$

$$+(1+\mu_{♄})\left\{\begin{array}{l}
+\ 0.00310 \\
+\ 0.00275 \text{ cos. } (♄-♆) \\
+\ 0.00003 \text{ cos. } 2\,(♄-♆)
\end{array}\right\}$$

$$+(1+\mu_{♄})\left\{\begin{array}{l}
-\ 0.00002 \text{ cos. } (♄-\pi_{♆}) \\
+\ 0.00002 \text{ cos. } (♄-2\,♆+\pi_{♆}) \\
-\ 0.0000002513\ t \text{ sin. } (♆-\pi_{♆})
\end{array}\right\}$$

$$+(1+\mu_{♄})\left\{\begin{array}{l}
-\ 0.00004 \text{ cos. } (♄-2\,♆+\pi_{♄}) \\
+\ 0.00008 \text{ cos. } (2\,♄-♆-\pi_{♄}) \\
+\ 0.0000006742\ t \text{ sin. } (♆-\pi_{♄})
\end{array}\right\}$$

$$+(1+\mu_{♃})\left\{\begin{array}{l} +\,0.00978 \\ +\,0.00497 \text{ cos. } (♃ - ♆) \end{array}\right\}$$

$$+(1+\mu_{♃})\left\{\begin{array}{l} -\,0.00004 \text{ cos. } (♃ - \pi_{♆}) \\ +\,0.00004 \text{ cos. } (♃ - 2\,♆ - \pi_{♆}) \\ -\,0.0000002163\, t \text{ sin. } (♆ - \pi_{♆}) \end{array}\right\}$$

$$+(1+\mu_{♃})\left\{\begin{array}{l} +\,0.00012 \text{ cos. } (2\,♃ - ♆ - \pi_{♃}) \\ +\,0.0000002751\, t \text{ sin. } (♆ - \pi_{♃}) \end{array}\right\}$$

$$+\,(1+\mu_{♂,\,⊕,\,♀,\,☿})\ 0.00006.$$

"These formulæ may be exhibited in the following form, which is similar to that adopted by the illustrious Leverrier, in his theory of Mercury, and very convenient for the construction of tables. The coefficients for the corrections of the masses are hereafter omitted.

$$\begin{array}{rl} \delta v = & -\ 8''.63 \text{ sin. } 2\,(♅ - ♆) \\ & -\ 1.70 \text{ sin. } 3\,(♅ - ♆) \\ & -\ 0.54 \text{ sin. } 4\,(♅ - ♆) \\ & -\ 0.21 \text{ sin. } 5\,(♅ - ♆) \\ & -\ 0.09 \text{ sin. } 6\,(♅ - ♆) \\ & -\ 0.04 \text{ sin. } 7\,(♅ - ♆) \\ & -\ 0.02 \text{ sin. } 8\,(♅ - ♆) \\ & -\ 0.01 \text{ sin. } 9\,(♅ - ♆) \\ & -\ 0.01 \text{ sin. } 10\,(♅ - ♆) \\ & +\ 18.60 \text{ sin. } (♄ - ♆) \\ & -\ 0.15 \text{ sin. } 2\,(♄ - ♆) \\ & -\ 0.02 \text{ sin. } 3\,(♄ - ♆) \\ & +\ 34.09 \text{ sin. } (♃ - ♆) \\ & -\ 0.02 \text{ sin. } 2\,(♃ - ♆) \\ & +\ 0.02 \text{ sin. } (⊕ - ♆) \\ & +\ k \text{ sin. } (♆ + \varphi) + A. \end{array}$$

$\delta r =$ 0.01365
+ 0.00097 cos. 2 (♅ — ♆)
+ 0.00023 cos. 3 (♅ — ♆)
+ 0.00008 cos. 4 (♅ — ♆)
+ 0.00003 cos. 5 (♅ — ♆)
+ 0.00001 cos. 6 (♅ — ♆)
+ 0.00001 cos. 7 (♅ — ♆)
+ 0.00275 cos. (♄ — ♆)
+ 0.00003 cos. 2 (♄ — ♆)
+ 0.00497 cos. (♃ — ♆)
+ h cos. (♆ — θ) + B.

"In these equations, k, h, φ, θ, A, and B are determined by the following formulæ: —

k cos. φ = 1″.87 cos. (♅ — ♆ + 306° 5′ 13″)
+ 0.22 cos. [2 (♅ — ♆) + 171 28]
+ 1.76 cos. [3 (♅ — ♆) + 0 5]
+ 0.35 cos. [4 (♅ — ♆) + 221 30]
+ 0.13 cos. 5 (♅ — ♆)
+ 0.07 cos. 6 (♅ — ♆)
+ 0.03 cos. [7 (♅ — ♆) + 348]
+ 0.01 cos. [8 (♅ — ♆) + 348]
+ 0.44 cos. (♄ — ♆ + 242 47)
+ 0.55 cos. [2 (♄ — ♆) + 271 3]
+ 0.44 cos. (♃ — ♆ + 208 18)
+ 0.82 cos. [2 (♃ — ♆) + 348 9]

k sin. φ = 1″.87 sin. (♅ — ♆ + 306° 5′ 13″)
+ 0.22 sin. [2 (♅ — ♆) + 171 28]
+ 1.70 sin. [3 (♅ — ♆) + 178 45]
+ 0.32 sin. [4 (♅ — ♆) + 180]
+ 0.11 sin. [5 (♅ — ♆) + 180]
+ 0.05 sin. [6 (♅ — ♆) + 180]
+ 0.03 sin. [7 (♅ — ♆) + 168]
+ 0.01 sin. [8 (♅ — ♆) + 168]
+ 0.51 sin. (♄ — ♆ + 77 0)
+ 0.55 sin. [2 (♄ — ♆) + 268 55]
+ 0.46 sin. [♃ — ♆ + 64 0]
+ 0.82 sin. [2 (♃ — ♆) + 348 9]

$$\begin{aligned}
h \cos.\,\theta = {} & 0.00011 \cos.\,(\text{♅} - \text{♆} + 121^{\circ}) \\
& + 0.00001 \cos.\,[2(\text{♅} - \text{♆}) + 252] \\
& + 0.00015 \cos.\,[3(\text{♅} - \text{♆}) + 333] \\
& + 0.00004 \cos.\,[4(\text{♅} - \text{♆}) + 333] \\
& + 0.00002 \cos.\,[5(\text{♅} - \text{♆}) + 348] \\
& + 0.00001 \cos.\,[6(\text{♅} - \text{♆}) + 348] \\
& + 0.00001 \cos.\,(\text{♄} - \text{♆} + 270) \\
& + 0.00008 \cos.\,[2(\text{♄} - \text{♆}) + 270] \\
& + 0.00005 \cos.\,(\text{♃} - \text{♆} + 90) \\
& + 0.00002 \cos.\,[2(\text{♃} - \text{♆}) + 348]
\end{aligned}$$

$$\begin{aligned}
h \sin.\,\theta = {} & 0.00011 \sin.\,(\text{♅} - \text{♆} + 301^{\circ}) \\
& + 0.00001 \sin.\,[2(\text{♅} - \text{♆}) + 72] \\
& + 0.00015 \sin.\,[3(\text{♅} - \text{♆}) + 333] \\
& + 0.00003 \sin.\,[4(\text{♅} - \text{♆}) + 333] \\
& + 0.00002 \sin.\,[5(\text{♅} - \text{♆}) + 348] \\
& + 0.00001 \sin.\,[6(\text{♅} - \text{♆}) + 348] \\
& + 0.00005 \sin.\,(\text{♄} - \text{♆} + 325) \\
& + 0.00008 \sin.\,[2(\text{♄} - \text{♆}) + 90] \\
& + 0.00005 \sin.\,(\text{♃} - \text{♆}) \\
& + 0.00012 \sin.\,[2(\text{♃} - \text{♆}) + 168]
\end{aligned}$$

$$\begin{aligned}
A = {} & 0.022332\, t \sin.\,(\text{♆} + 284^{\circ}\,56'\,38'') \\
& + 187.33 \sin.\,(\text{♅} - \text{♆} + 38\;24\;43) \\
& + 1724.04 \sin.\,(2\,\text{♆} - \text{♅} + 188\;42\;10) \\
& + 67.93 \sin.\,(2\,\text{♅} - 3\,\text{♆} + 358\;40\;\;0) \\
& + 14.45 \sin.\,(3\,\text{♆} - \text{♅} + 140\;55\;30) \\
& + 0.15 \sin.\,(4\,\text{♆} - \text{♅} + 90\;\;0) \\
& + 0.14 \sin.\,(\text{♅} + 257\;55) \\
& + 73.23 \sin.\,(4\,\text{♆} - 2\,\text{♅} + 181\;20\;17) \\
& + 0.59 \sin.\,(5\,\text{♆} - 2\,\text{♅} + 134\;19) \\
& + 2.85 \sin.\,(6\,\text{♆} - 3\,\text{♅} + 87\;47\;23) \\
& + 5.66 \sin.\,(3\,\text{♅} - 5\,\text{♆} + 358\;16\;47) \\
& + 0.09 \sin.\,(2\,\text{♅} - 2\,\text{♆} + 300)
\end{aligned}$$

$$\begin{aligned}
B = {} & 0.0000030384\, t \sin.\,(\text{♆} + 257^{\circ}\,27'\,18'') \\
& + 0.01326 \cos.\,(\text{♅} - \text{♆} + 179\;56\;48) \\
& + 0.00696 \cos.\,(2\,\text{♆} - \text{♅} + 7\;51\;\;1) \\
& + 0.00462 \cos.\,(2\,\text{♅} - 2\,\text{♆} + 179\;\;7\;45) \\
& + 0.00005 \cos.\,(3\,\text{♆} - \text{♅} + 149) \\
& + 0.00116 \cos.\,(4\,\text{♆} - 2\,\text{♅} + 3\;29\;\;5)
\end{aligned}$$

" The terms *A* and *B* include the secular terms, and also those of long period as well as those which acquire large coefficients by the small divisors, which depend upon the near approach to commensurability in the mean motions of Neptune and Uranus. These coefficients will vary very sensibly by a change in the value of the mean motion of Neptune arising from a more accurate determination of its orbit. But the principal effect of these terms can for a limited period, such as a century, for instance, be included in the ordinary forms of elliptic motion, and the residual portion will assume a secular form, which is no more liable to change, from a new correction of the mean motion of Neptune, than the other small coefficients of the equations of perturbation. The elliptic portions of *A* and *B* may therefore be neglected until longer observation has given a more precise value of Neptune's mean motion, and the residual portion is contained in the following equation.

$$\begin{aligned}
A = {} & \, 1.98 \ \sin.\ (2\,\text{♅} + 2\overset{\circ}{1}1\ \overset{\prime}{4}6)^{\prime\prime} \\
& + 0.36517\ t \ \sin.\ (\text{♅} + 20\ 58\ 12) \\
& + 0.0002\ t \ \sin.\ (2\,\text{♅} + 322) \\
& - 0.0020714\ t^2 \\
& + 0.0004027\ t^2\ \sin.\ (\text{♅} + 230^\circ\ 3')
\end{aligned}$$

$$\begin{aligned}
B = {} & - 0.00066 \\
& + 0.00134 \ \cos.\ (\text{♅} + 38^\circ\ 40') \\
& + 0.0000115\ t \\
& + 0.00002827\ t \ \sin.\ (\text{♅} + 286^\circ\ 32'\ 23'') \\
& + 0.0000000300\ t^2\ \cos.\ (\text{♅} + 125\ \ 1)
\end{aligned}$$

" The following particular values of δv and δr, derived from the preceding formulæ, will be useful in computing the orbit of Neptune from observation.

	δv	δr
	″	
May 9, 1795,	+ 47.80	+ 0.01283
October 1, 1846,	+ 27.03	+ 0.01793
January 1, 1847,	+ 27.13	+ 0.01728
April 1, "	+ 26.68	+ 0.01664
July 1, "	+ 25.75	+ 0.01602
October 1, "	+ 24.37	+ 0.01544

January 1, 1848,	+ 22.58	+ 0.01491
April 1, “	+ 20.40	+ 0.01443
July 1, “	+ 17.89	+ 0.01400
October 1, “	+ 15.12	+ 0.01363
January 1, 1849,	+ 12.18	+ 0.01332
April 1, “	+ 9.06	+ 0.01308
July 1, “	+ 5.84	+ 0.01290
October 1, “	+ 2.59	+ 0.01277
January 1, 1850,	— 0.64	+ 0.01270
April 1, “	— 3.83	+ 0.01270
July 1, “	— 6.96	+ 0.01276
October 1, “	— 9.96	+ 0.01288
January 1, 1851,	— 12.64	+ 0.01308.”

Professor Peirce communicated the following elements of the orbit of the satellite of Neptune, computed from the combination of all of Lassell's and Mr. Bond's observations; and he also communicated the corresponding mass of the primary.

“ Time of sidereal revolution, 5 days 21 hours 12.4 minutes.

“ Inclination to ecliptic, 29°.9.

“ Longitude of ascending node (the motion being supposed direct), 119°.8.

“ Time of greatest northern elongation, November $26^{d}.53$, Greenwich mean solar time.

“ Greatest elongation, 16″.5.

“ Distance of satellite from Neptune, 230,000 miles.

“ Corresponding mass of Neptune, $\frac{1}{18780}$, the mass of sun being unity.

“ The time of sidereal revolution is not liable to an error of more than a few minutes, and the greatest elongation cannot be less than 16″.3, or more than 17″.0. The mass of Neptune, therefore, cannot be less than $\frac{1}{19500}$, or greater than $\frac{1}{17000}$.”